Is Rebalancing a Portfolio During Retirement Necessary?

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Executive Summary

- This paper investigates the strategy of rebalancing the retirement portfolio during the withdrawal phase. The goal is to provide the largest number of equal (real) withdrawals from a given retirement portfolio.
- The study investigates six different allocations of stock and five different harvesting rules, only one of which rebalances the portfolio annually. The methods are tested using five different withdrawals rates (3–7 percent). The results look at shortfalls over 30 years, as well as shorter periods.
- The study uses two analysis methods: bootstrap and historical inflation-adjusted rates of return in their true temporal order. Both methods find that rebalancing provides no significant protection on portfolio longevity, and this holds for all withdrawal periods. In fact, in some cases, rebalancing increases the number of shortfalls.
- Withdrawing bonds first, over stocks, performs the best of all the methods, though the resulting stock-heavy portfolio may make some investors uneasy. This method also is most apt to leave a larger remaining balance at the end of 30 years, while rebalancing leaves the smallest amount.
- Withdrawing stocks first leaves more shortfalls than withdrawing low first or high first.
- Confirming previous research, the larger the proportion of stocks to bonds, the longer the portfolio lasts; the higher the withdrawal rate, the more shortfalls.
- The results suggest that the use of life-cycle funds or a life-cycle strategy that decreases stock proportions as one grows older needs empirical justification.

When approaching investment objectives and policy for individual investors, many financial advisors take a life-cycle approach. Depending on the human-capital phase of life and the current level of financial wealth, investors are broadly classified in one of the four stages of the life cycle: accumulation, consolidation, spending (withdrawal), or gifting. Advisors provide guidance on risk tolerance and suitable investments based on this classification. For a succinct discussion on the investment implications of the four phases, see Reilly and Brown (2003).

Over the last 20 years or so, asset allocation decisions have been the centerpiece of most portfolio management discussions, both for academics and practitioners. The importance of the asset allocation decision in the accumulation process of wealth is widely agreed upon; consequently, most of the previous research has focused on this aspect of portfolio management.

With individuals living longer and with extended retirement periods, there is a greater need to study wealth management in the spending phase of the individual's life. The spending (withdrawal) phase of the life-cycle approach is the focus of this paper.

At the time of retirement, an individual investor faces a number of strategic investment decisions, one of which is whether rebalancing the portfolio should be continued during the withdrawal phase. Bootstrap simulations[1] using historical rates of return on U.S. stocks and U.S. Treasury

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bonds are used to observe the effect of depleting a retirement portfolio consisting of stocks and bonds. The following questions are answered:

1. Does rebalancing the portfolio at the end of every year decrease the likelihood of a shortfall or increase the likely balance remaining at the end of 30 years?
2. If the portfolio is *not* rebalanced, will depleting the higher-returning asset first make the money last longer? Or should the lower-returning asset be depleted first?
3. Do the answers to the previous questions in any way depend on asset allocation decisions? In other words, will different answers ensue if the allocation between stocks and bonds is 50/50, rather than 70/30, 60/40, 40/60, or 30/70?

The results provide guidance to individual investors and their advisors on wealth and asset allocation decisions during retirement. Some of the answers are surprising and counter to conventional advice.

Literature Review

Perhaps one of the first insights into an individual investor's life-cycle approach to investing was formulated by Modigliani and Brumberg (1954), who postulated that an individual attempts to maximize utility (current consumption), where consumption ability is based on the consumer's wealth and the return on capital. Most of life-cycle research has been focused on the accumulation phase. See for example, Dammon, Spatt, and Zhang (2004); Malkiel (1999); Jagannathan and Kocherlakota (1996); Jones and Wilson (1999); and Arshanapalli et al. (2001). Recently, attention has been focused on the spending phase of the life cycle. Milevsky, Kwok, and Robinson (1994, 1997) use Canadian mortality tables and asset class returns to show that an optimal asset allocation during retirement is 75 to 100 percent in equities. Drawing from research in the area of dollar-cost averaging, Vora and McGinnis (2000) show that a 100 percent stock portfolio resulted in a higher withdrawal rate during retirement compared with a 100 percent bond portfolio. Using historical rates of returns on asset classes, Cooley, Hubbard, and Walz (1999) demonstrate that a portfolio invested 75 percent in large-cap U.S. stocks can be subject to an inflation-adjusted sustainable withdrawal rate of 4 to 5 percent. Bengen (2004) argues that inflation-adjusted withdrawals of 4 percent of the starting portfolio balance are sustainable over at least a 30-year period with a 50–50 stock-bond allocation. In a recent paper, Spitzer and Singh (2006) show that portfolio longevity is independent of the tax status of the subportfolios constituting it. They also demonstrate that portfolio longevity can be extended by withdrawing first from a subportfolio with a lower expected return and subsequently from one with a higher expected return.

In the sections below, emphasis is on the spending phase of the life cycle; comparisons of how long a portfolio lasts before running out of money when inflation-adjusted withdrawals of a constant size are made. Historical U.S. data are used to compare withdrawal strategies that rebalance the portfolio post-withdrawal versus withdrawal processes that do not rebalance (akin to "buy and hold"). Statistical results for both a bootstrap analysis and an analysis that uses the historical data in its chronological order are also presented.

Does Rebalancing Matter?

Rebalancing the investment portfolio during the accumulation phase is a recommended practice among financial advisors. In fact, many investment managers, such as TIAA-CREF, now offer automatic periodic rebalancing of client portfolios as an additional service. When rates of return vary, the way in which money is withdrawn may affect the longevity of the withdrawal process. Two questions are the focus of this study: (1) How does rebalancing affect portfolio longevity? and (2) If the portfolio is not rebalanced, does the order in which assets are withdrawn affect longevity? The manner in which these questions will be addressed relies on the conditions, data, variables, and notation described next.

Asset Allocation, Withdrawal Amounts, and Withdrawal Methods

Six different stock/bond mixes will be investigated: 30/70, 40/60, 50/50, 60/40, 70/30, and 80/20.

For each of these allocation mixes and each of five withdrawal amounts, there will be five ways in which money is withdrawn from the portfolio.[2]

1. Withdraw money from either stocks or bonds and then rebalance the portfolio annually to the initial stock/bond proportion. This harvesting rule will be referred to as "Rebalance."
2. Withdraw money from the asset that had the *highest* return during the year and do not rebalance. This will be referred to as "High First."
3. Withdraw money from the asset that had the *lowest* return during the year and do not rebalance. This will be referred to as "Low First."

   To the extent that historical rates of return on bonds tend to be lower than historical rates of return on stocks, the following two additional methods of harvesting withdrawals will be referred to as "Bonds First" and "Stocks First."
4. Take withdrawals from bonds first and do not rebalance.
5. Take withdrawals from stocks first and do not rebalance.

The longevity of the withdrawal process clearly depends on how cautious or aggressive the investor is in terms of the amount of money withdrawn. The study will look at five levels of withdrawal: 3, 4, 5, 6, or 7 percent of the initial portfolio balance. This should represent a cautious (3 percent) to a very aggressive (7 percent) withdrawal pattern.

There are six initial stock/bond proportions times five withdrawal percentages times five harvesting methods for a total of 150 conditions. For each of these 150 conditions, the number of times the portfolio runs out of money before 30 years of withdrawals have taken place will be calculated. The smaller this count is, the more successful the withdrawal strategy.

Taxes are ignored in this study. The fundamental question being asked is "Does rebalancing during retirement extend the life of the portfolio?" Two types of portfolios can easily be addressed where taxes will have no effect on the answer to the question: (1) tax-deferred accounts where all withdrawals are taxed at the same rate regardless of portfolio composition, or (2) Roth IRAs, which are not taxed at all. In the first instance, if bond earnings and stock earnings are taxed equivalently, then whether bonds or stocks are harvested first will have little effect on how long the portfolio lasts since there is no preferential tax treatment. In the second instance, withdrawals from the Roth IRA have no tax consequences.

Data, Variables, and Notation

Annual inflation-adjusted rates of return from 1926 through 2003 for stocks (S&P 500) and bonds (long-term U.S. Treasury bonds) were obtained from *Stocks, Bonds, Bills and Inflation*, EnCorr Database, 2004 Edition, Ibbotson Associates. The following variables are used to define the model and to describe the estimation process:

- t = the year in which the withdrawal occurs; $t = 1, 2, \ldots T$.
- W_0 = \$100, the starting amount of wealth, beginning in year 0.
- γ = the withdrawal percentage (3, 4, 5, 6, and 7 percent).
- c = γW_0 = the withdrawal at the end of each year (\$3, \$4, \$5, \$6, or \$7), inclusive of any taxes that may be due.
- r_{st} = annual inflation-adjusted rate of return on stocks at period t.
- r_{bt} = annual inflation-adjusted rate of return on bonds at period t.
- λ = the proportion of the portfolio designated for stocks. Allocations are made only between stocks and bonds. For example, a λ of 0.30 means that 30 percent of the portfolio is allocated to stocks and 70 percent is allocated to bonds.

Rates of return, r_{st} and r_{bt}, vary with t. The starting amount, W_0 = \$100, is arbitrary. All results will be presented as percentages and are therefore not dependent on the actual dollar amount. Presentation in this format is independent of portfolio size, and results are equally applicable to the \$100,000 client or the \$10 million client. The withdrawal amounts, c, are all in real (inflation-adjusted) terms and do not change during the simulation. For example, if 4 percent of the starting balance of \$100 is withdrawn, then c = \$4 will be withdrawn each year. The \$4 represents 4 percent of the starting balance in real terms, that is, after any adjustment for inflation.

$W_{s0} = \lambda W_0$ is the starting amount of the portfolio allocated to stocks, (1)

$W_{b0} = (1 - \lambda)W_0$ is the starting amount of the portfolio allocated to bonds, (2)

For each year $t = 1, 2, \ldots T$

$$S = W_{st}(1 + r_{st}), \quad (3)$$

is the value of the stock portfolio before rebalancing at end of year t and

$$B = W_{bt}(1 + r_{bt}) \quad (4)$$

is the value of the bond portfolio before rebalancing at end of year t.

The harvesting methods are High First, Low First, Bonds First, or Stocks First. The Low First is different from Bonds First. For Low First, whichever return (stock or bond) is smallest in any given year will be harvested first. An example of High First is shown below.

High First harvest. If $r_{st} \geq r_{bt}$,

$W_{st} = S - c$ and $W_{bt} = B$.
[Harvest stocks first if highest return]

Otherwise,

$W_{bt} = B - c$ and $W_{st} = S$.
[Harvest bonds first if highest return] (5)

If either W_{bt} or W_{st} is negative, the shortfall in the withdrawal is taken from the other asset and the asset with the negative value is set to zero.

Bonds First harvest. An example of Bonds First harvest is shown as

$W_{bt} = B - c$ and $W_{st} = S$;

that is, take the withdrawal from bonds. A parallel construct is easily derived for Stocks First.

If either W_{bt} or W_{st} is negative, the shortfall in the withdrawal is taken from the other asset and the asset with the negative value is set to zero.

In the absence of rebalancing, the asset allocation will change over time when using any of the four harvesting methods. Bonds First and Low First will tend to increase the proportion of stock over time, while Stocks First and High First will tend to decrease the proportion of stocks over time.

Rebalancing. If rebalancing occurs, then

$$W_{st} = \lambda(S + B) \text{ and}$$
$$W_{bt} = (1 - \lambda)(S + B) \quad (6)$$

The next section presents the statistical framework and results of the study.

Results

This section is broken into two major parts: (1) the shortfall analyses (there are two shortfall analyses) and (2) the average-balance-remaining analysis. The shortfall analyses find the relative frequency of running out of money before a period of 30 years has elapsed under the 150 conditions described above. The *first* shortfall analysis uses a bootstrap method, fully described below, followed by the results of that analysis. The *second* shortfall analysis, which will be referred to as "temporal order," uses historical inflation-adjusted (real) rates of return *in their true temporal order*. Using rolling 30-year periods in this manner preserves any serial correlations within the data. A description of the temporal order method and its results follow. The second major part of this section uses data obtained from the bootstrap method to measure the size of the average balance remaining (if any) in the portfolio after 30 years.

Shortfall Analyses

Bootstrap algorithm. The following steps are repeated 10,000 times for each of the six λs, for each of the five harvesting strategies, and for each of the five different values of the withdrawal amounts *c*. The values for *c* (*c* = \$3, \$4, \$5, \$6, or \$7) are the real (inflation-adjusted) withdrawals representing a range of 3 to 7 percent of the starting balance.

Set $t = 1$, $W_0 = \$100$, T = 30. Select the value of *c*.

(a) Randomly generate a number between 1926 and 2003 (inclusive), which is the "current year" subscript. Obtain r_b and r_s for this "year." (This retains the asset class cross-correlations.)

(b) Compute W_{st} and W_{bt} of equation (5) or equation (6) as appropriate.

(c) Increment *t*. If $t > T$, save the value of $(W_{st} + W_{bt})$ for later analysis; otherwise, go to step (a). If the portfolio has no money left, a count of a shortfall for that set of conditions is incremented.

Steps (a), (b), and (c) constitute a single iteration of the bootstrap. There will be 10,000 such iterations for each of the 150 conditions.

Bootstrap results. Values of the "percent of times (out of 10,000) that the portfolio ran out of money before 30 years" are presented in Table 1 for each of the 150 – γ, λ, and harvest conditions. While certain patterns are easily discerned (such as shortfalls increase with the withdrawal amount – γ; shortfalls decrease with the proportion of the portfolio devoted to stocks – λ), there are difficulties in trying to ascertain the answer to the central question: "Is there an optimal strategy for harvesting and does it depend on rebalancing?"

To avoid constructing a plethora of tables and performing numerous pair-wise t-tests, the following synthesis is offered. A multiple linear regression equation will be specified and estimated that uses dummy variables to find the harvesting method with the fewest shortfalls while controlling for the changing stock/bond composition and the changing withdrawal amounts. The following variables are used in the regression equation:

- *Y* = The percentage of shortfalls (out of 10,000) in any set of conditions below
- γ_3 = 1 when the withdrawal rate, γ = 3%, 0 otherwise
- γ_5 = 1 when γ = 5%, 0 otherwise
- γ_6 = 1 when γ = 6%, 0 otherwise
- γ_7 = 1 when γ = 7%, 0 otherwise
- λ_3 = 1 when the stock proportion, λ = 0.3, 0 otherwise
- λ_4 = 1 when λ = 0.4, 0 otherwise
- λ_6 = 1 when λ = 0.6, 0 otherwise
- λ_7 = 1 when λ = 0.7, 0 otherwise
- λ_8 = 1 when λ = 0.8, 0 otherwise
- R = 1 when Rebalance, 0 otherwise
- BF = 1 when Bonds First, 0 otherwise
- SF = 1 when Stocks First, 0 otherwise
- HF = 1 when High First, 0 otherwise

There are 14 independent variables in the regression, including the intercept. Note that the interpretation of the regression equation will be relative to Low First, $\lambda = 0.5$, and $\gamma = 4\%$; those values are *omitted* from the regression equation. When all dummy variables are zero, the value of *Y* will be the intercept alone; that is, the percent of shortfalls at $\lambda = 0.5$, $\gamma = 4\%$ with Low First harvesting. When a dummy variable is activated (equals 1), it will change the value of *Y* by the estimated value of the regression coefficient on that dummy variable. The regression equation can be written as

$$Y = \beta_0 + \beta_1\gamma_3 + \beta_2\gamma_5 + \beta_3\gamma_6 + \beta_4\gamma_7 + \beta_5\lambda_3 + \beta_6\lambda_4 + \beta_7\lambda_6 + \beta_8\lambda_7 + \beta_9\lambda_8 + \beta_{10}R + \beta_{11}BF + \beta_{12}SF + \beta_{13}HF \quad (7)$$

So, for example, β_0 is the approximate percentage of run-outs when $\lambda = 0.5$, withdrawals are at 4 percent, and Low First. If the stock percentage is decreased to $\lambda = 0.3$, then the approximate percentage of run-outs is given by $\beta_0 + \beta_5$. Any positive coefficient (β) indicates an increase in the number of shortfalls, while any negative coefficient shows a decrease in shortfalls.

Striking Observations

The regression results for the bootstrap are provided in Table 2 and indicate an excellent fit of the data. The percentage of the variation in *Y* explained by the set of dummy variables (R^2) is approximately 93 percent.

1. One of the most striking observations is that Rebalance (R) is *not* significant. That is, shortfall remains unaffected by rebalancing when holding other variables constant.
2. The percent of shortfalls for withdrawal rates of 3 percent ($\gamma = 3\%$) is statistically different from the percent of shortfalls for $\gamma = 4\%$, as are withdrawal rates of 5, 6, and 7 percent. A withdrawal rate of 3 percent results in 8.54 percent fewer shortfalls than a withdrawal rate of 4 percent, while a withdrawal rate of 7 percent, for example, increases the shortfall percentage by 46 percent.
3. The coefficients on γ_5, γ_6, and γ_7 are all significant and positive, indicating that the percent of shortfalls increases as γ (the withdrawal rate) increases above 4 percent.
4. The coefficients on λ_3 and λ_4 (stock proportions of 30 and 40 percent, respectively) are statistically significant and positive, indicating that the percent of shortfalls *increases* as the proportion allocated to stocks, λ, decreases from 0.5. The percentage of shortfalls for λ_6 is not statistically different from $\lambda = 0.5$, but the coefficients on λ_7 and λ_8 are both different

from zero and negative, indicating that as the proportion of stock in the portfolio increases, the percentage of shortfall tends to decrease. For example, other things held constant, the percent of shortfalls for $\lambda = 0.7$ is about 3.76 percent less than for $\lambda = 0.5$, while the percent of shortfalls for $\lambda = 0.3$ is about 8.8 percent greater than for $\lambda = 0.5$.

5. Bonds First (BF) is significant and results in about 4.4 percent fewer shortfalls than Low First, the baseline reference.
6. Stocks First (SF) is significant and positive; harvesting Stocks First results in about 9 percent *more* shortfalls than Low First. Withdrawing stocks first, irrespective of last year's performance, results in higher shortfalls, all else held constant. Lastly, High First was not significant; that is, it did not perform differently from Low First.

Some readers might have concerns that the probability of shortfall over 30 years does not provide information about when the shortfalls occur in the 30-year span. For example, a shortfall in the 15th year might be viewed as more harmful than a shortfall in the 29th year. To address this concern, the number of shortfalls that occurred in each year for all γ, λ, and models was tabulated during the simulation process. The amount of information

Table 1: Percentage of Times Under Bootstrap Simulation Portfolio Was Exhausted Before 30 Years

The percentage of times out of 10,000 (bootstrap simulation) that the portfolio was exhausted before 30 years, for six "Stock Percentage of Portfolio," for five rates of withdrawal (γ), for "Rebalance" and "No Rebalance for four different harvesting methods."

Stock Percentage of Portfolio	Withdrawal Rate	Percentage of Shortfalls	Percentage of Shortfalls			
		Rebalance	No Rebalance Methods			
	γ		High First	Low First	Bonds First	Stocks First
30	3%	1	3	3	2	4
	4%	9	15	14	8	21
	5%	30	39	34	22	49
	6%	55	64	56	38	74
	7%	75	82	71	55	89
	γ		High First	Low First	Bonds First	Stocks First
40	3%	1	2	3	2	3
	4%	8	11	12	9	18
	5%	24	29	28	21	42
	6%	45	51	46	35	65
	7%	67	70	61	50	83
	γ		High First	Low First	Bonds First	Stocks First
50	3%	1	2	3	2	3
	4%	8	9	11	9	16
	5%	20	23	25	20	36
	6%	41	41	41	32	57
	7%	58	56	55	46	74
	γ		High First	Low First	Bonds First	Stocks First
60	3%	2	2	3	3	3
	4%	8	9	11	10	15
	5%	20	21	23	20	31
	6%	36	34	36	33	50
	7%	53	49	50	45	67
	γ		High First	Low First	Bonds First	Stocks First
70	3%	2	3	3	3	4
	4%	8	9	11	10	13
	5%	19	20	22	20	29
	6%	34	33	34	33	45
	7%	49	45	46	44	60
	γ		High First	Low First	Bonds First	Stocks First
80	3%	3	4	4	5	4
	4%	9	11	12	11	14
	5%	20	21	21	21	26
	6%	33	32	33	33	40
	7%	46	42	45	43	53

Table 2: Shortfall Regression Results for Bootstrap—Regression of 'Percent of Shortfalls' on Multiple Dummy Variables

	Bootstrap Regression	
	Coefficients	P-Value
Intercept	10.13	0.0000
γ_3	−8.54	0.0000
γ_5	14.58	0.0000
γ_6	31.42	0.0000
γ_7	46.41	0.0000
λ_3	8.80	0.0000
λ_4	3.72	0.0274
λ_6	−2.32	0.1663
λ_7	−3.76	0.0257
λ_8	−4.24	0.0121
R	−1.06	0.4878
BF	−4.44	0.0042
SF	8.97	0.0000
HF	0.53	0.7292

Number of Observations = 150
$R^2 = .93$ SEE = 58.97

gathered is too voluminous to present, but for the particular (and arguably "most practical") cases of (1) 4 percent withdrawals, with 50/50 stock/bonds and (2) 4 percent withdrawals with 60/40 stock/bonds, the shortfall frequency pattern by year for the Rebalance model and the Bonds First model were trivially different. *Shortfalls did not occur earlier when there was no rebalancing.* In both cases, the timing of the shortfalls was quite similar; the earliest shortfalls occurred at year 12, with the frequency of shortfalls increasing over time. The risks appear to be very much the same for these cases.

From these "shortfall by year" data, it is also possible to explore whether the conclusions listed above hold for withdrawal horizons shorter than 30 years. The shortfall regressions were re-estimated using percentage of shortfall after 15, 20, and 25 years as dependent variables. The results of these regressions are not presented here but are available from the authors upon request. The following conclusions can be stated: For each of the three shorter duration withdrawals, *Rebalance is never significant and Bonds First always results in significantly fewer shortfalls.*

Table 3: Percentage of Times Under Temporal Order Portfolio Was Exhausted Before 30 Years

The percentage of times (out of 49 trials) that the portfolio was exhausted before 30 years, for six "Stock Percentage of Portfolio" for five rates of withdrawal, for "Rebalance" and "No Rebalance for Four Different Harvesting Methods."

Stock Percentage of Portfolio	Withdrawal Rate	Percentage of Shortfalls	Percentage of Shortfalls			
		Rebalance	No Rebalance Methods			
	γ		High First	Low First	Bonds First	Stocks First
30	3%	0	0	0	0	6
	4%	22	43	33	10	57
	5%	69	76	55	37	92
	6%	92	94	71	43	98
	7%	96	100	84	63	100
	γ		High First	Low First	Bonds First	Stocks First
40	3%	0	0	0	0	2
	4%	14	20	16	4	47
	5%	51	55	45	29	71
	6%	82	76	59	43	92
	7%	94	92	73	55	100
	γ		High First	Low First	Bonds First	Stocks First
50	3%	0	0	0	0	2
	4%	10	10	10	4	37
	5%	37	39	41	27	67
	6%	59	61	53	41	78
	7%	86	73	61	51	94
	γ		High First	Low First	Bonds First	Stocks First
60	3%	0	0	0	0	0
	4%	6	4	8	4	22
	5%	35	29	33	18	55
	6%	55	45	47	41	69
	7%	69	61	61	53	82
	γ		High First	Low First	Bonds First	Stocks First
70	3%	0	0	0	0	0
	4%	4	2	4	2	22
	5%	29	22	27	16	49
	6%	47	43	47	41	57
	7%	63	57	53	51	71
	γ		High First	Low First	Bonds First	Stocks First
80	3%	0	0	0	0	0
	4%	2	2	2	2	14
	5%	22	20	24	22	37
	6%	43	41	41	39	53
	7%	57	49	51	51	65

Temporal Order Analysis Using Historical Sequences

Rather than select the rates of return for stocks and bonds in a random order, this section looks at all possible 30-year sequences and counts the number of times shortfalls occur. There are 49 overlapping periods of 30 years in the annual data from 1926 to 2003: for example, 1926–1955, 1927–1956…1974–2003. For each of these 49 periods, the count of the number of times that the investor ran out of money is calculated. The same 150 conditions are used in this part of the study as in the previous part; the difference is that the 30-year sequences are historical and not randomly selected. This methodology ensures that both serial- and cross-correlations that existed during the 30-year sequences have been retained.[3]

Shortfall analysis: temporal order results. The percent of shortfalls (out of 49) for each of the 150 conditions is shown in Table 3. The identical multiple linear regression equation used earlier is re-employed here and is shown in Table 4. While there still are 150 different conditions, there are only 49 possible sequences and not 10,000 replications as in the preceding section. An excellent fit of the data, $R^2 = 0.92$, is obtained. Many conclusions reached here are identical to the conclusions reached for the bootstrap analysis, including the conclusion concerning the presumed benefit of rebalancing.

1. Rebalance (R) is significant ($p < .05$), but not in a constructive way. Rebalancing has a positive coefficient so that the percent of shortfalls increase by about 4.8 percent when it is active.
2. The coefficients on all included γs are statistically different from the percent of shortfall for $\gamma = 4\%$. Like the bootstrap, γ_3 results in significantly fewer shortfalls (–14 percent) while γs greater than 4 percent, γ_5, γ_6, and γ_7, are all significant and positive, indicating that the percent of shortfalls increases as γ (withdrawal rate) increases.
3. The coefficients on the proportion of wealth invested in stocks, λ_6, λ_7, and λ_8, are statistically significant and negative, indicating that the percent of shortfalls *decreases* as the proportion allocated to stocks, λ, increases. For stock proportions less than 0.5, the percentage of shortfalls increases.
4. Stocks First is statistically significant, but in a deleterious manner: if stocks are harvested first, the percent of shortfalls increases by about 18 percent.
5. High First is not statistically different from Low First.

Table 4: Shortfall Regression Results: Temporal Order—Regression of 'Percent of Shortfalls' on Multiple Dummy Variables

	Temporal Order Regression	
	Coefficients	P-Value
Intercept	11.70	0.0000
γ_3	–14.35	0.0000
γ_5	26.26	0.0000
γ_6	43.61	0.0000
γ_7	55.92	0.0000
λ_3	16.00	0.0000
λ_4	7.18	0.0058
λ_6	–5.71	0.0274
λ_7	–9.31	0.0004
λ_8	–12.08	0.0000
R	4.83	0.0408
BF	–8.44	0.0004
SF	18.03	0.0000
HF	3.81	0.1057

Number of Observations = 150
$R^2 = .92$ SEE = 9.06

Balance-Remaining Analysis with the Bootstrap

Both preceding "shortfall" results are based on simple counts of failure to attain the 30-year withdrawal time span. An alternative measure of the utility of rebalancing during the withdrawal phase is based on success rather than failure: How much money is left over from the original $100 after 30 years have elapsed under these various withdrawal strategies? In a sense, the question is akin to asking which strategies will provide a larger inheritance. Table 5 shows the percent of initial balance remaining (PIBR) that remains after 30 years for each of the 150 different conditions. The first number in the table, 206, (for Rebalance, $\lambda = 0.3$, $\gamma = 3\%$), indicates that the average ending balance is 206 percent of the starting balance (2.06 times the initial portfolio). The numbers in Table 5 were averaged[4] over the 10,000 iterations in the bootstrap analysis.

In Table 5, the PIBR increases with λ, the stock proportion, for any given withdrawal rate or harvesting strategy. For any λ and γ, Bonds First always provides a higher PIBR than Stocks First and Low First provides a higher PIBR than High First. Lastly, both Bonds First and Low First provide a larger PIBR than Rebalance, for any λ and γ pair. The evidence suggests that rebalancing is not a good strategy if building an estate is a priority.

Table 6 uses the same regression analysis as previously used in an attempt to determine if the suggestions of Table 5 are statistically significant.

The PIBR is the dependent variable and all the independent variables retain their previous meaning. The results in Table 6 have a very high R^2 of 0.97. While many conclusions are similar to those of the shortfall analysis, several conclusions about harvesting and rebalancing are much more definitive.

1. Rebalance (R) has a significant negative effect on the PIBR. The PIBR is 444 for a 50/50 stock/bond allocation with 4 percent withdrawal rate, using the Low First harvesting method. (The value of 444 is the intercept value in the regression equation.) If Rebalance is employed rather than Low First, the PIBR declines by 169. Instead of having an ending balance that was 4.44 times the starting balance, the ending balance with Rebalance is only 2.75—a sizable effect.
2. The coefficients on γ_5, γ_6, and γ_7 are all significant and negative, indicating that the PIBR decreases as γ increases. Somewhat obviously, the smaller the amount withdrawn the larger the average ending balance.
3. As λ—the proportion of wealth invested in equities—increases, so does the PIBR.
4. Bonds First (BF) significantly increases PIBR over Low First.
5. Stocks First (SF) is a poor strategy since PIBR falls by more than 158 percent if stocks are harvested first.
6. The difference between harvesting Bonds First rather than Stocks First is quite large. Bonds First increases PIBR by about 72 percent while Stocks First decreases PIBR by more than 158 percent. The combined effect will make a large difference in PIBR. This outcome makes sense since the traditionally higher-earning asset (equities) is retained for a longer time in the portfolio if bonds are harvested first. The property is consistent with the findings of Spitzer and Singh (2006), who showed that withdrawing money from bonds (the historically lower rate of return asset) first and then withdraw-

ing from stocks, allowed the retirement portfolio to last longer.

7. High First is significantly worse than Low First, resulting in a decrease of almost 63 percent in PIBR.

The PIBR regression analysis strongly suggests that harvesting Bonds First and *not* rebalancing will provide the largest remaining balance. This conclusion is in agreement with the shortfall analysis in Table 2: Bonds First decreases shortfalls; Stocks First increases shortfalls. The PIBR analysis suggests that when a harvesting strategy does not fail (that is, the portfolio does not run out of money), the Bonds First harvesting method provides the largest balance remaining after 30 years.

Table 5: Average 'Balance Remaining' as a Percentage of the Starting Balance at the End of 30 Years

Average "Balance Remaining" as a percentage of the Starting Balance at the end of 30 years, for six "Stock Percentage of Portfolio," for five rates of withdrawal, for "Rebalance" and "No Rebalance for four different harvesting methods."

Stock Percentage of Portfolio	Withdrawal Rate	Percent of Balance Remaining	Percent of Balance Remaining			
		Rebalance	No Rebalance Methods			
	γ		High First	Low First	Bonds First	Stocks First
30	3%	206	260	318	420	183
	4%	145	170	242	366	108
	5%	93	96	176	294	53
	6%	51	47	130	227	22
	7%	24	19	95	168	9
	γ		High First	Low First	Bonds First	Stocks First
40	3%	259	368	424	524	282
	4%	192	265	347	458	177
	5%	132	185	279	386	97
	6%	83	117	214	309	50
	7%	42	63	159	244	24
	γ		High First	Low First	Bonds First	Stocks First
50	3%	320	479	522	611	377
	4%	249	365	469	558	261
	5%	184	282	367	454	177
	6%	121	199	286	372	100
	7%	77	142	218	299	60
	γ		High First	Low First	Bonds First	Stocks First
60	3%	408	582	631	722	473
	4%	323	496	553	629	361
	5%	244	384	468	543	253
	6%	176	302	365	425	178
	7%	114	223	284	343	111
	γ		High First	Low First	Bonds First	Stocks First
70	3%	506	688	739	823	580
	4%	404	583	631	710	449
	5%	317	483	530	594	349
	6%	236	395	445	463	260
	7%	172	309	346	380	187
	γ		High First	Low First	Bonds First	Stocks First
80	3%	620	777	819	835	690
	4%	528	667	712	727	557
	5%	397	551	592	617	450
	6%	328	475	509	500	344
	7%	242	379	384	402	267

Conclusions

While the wisdom of rebalancing in the accumulation phase of the life cycle is widely accepted, the wisdom does not appear to extend to the withdrawal phase. In both the bootstrap analysis and the temporal order analysis,

1. Rebalancing during the withdrawal phase provides no significant protection on portfolio longevity. This conclusion appears to hold for withdrawal periods of 15, 20, 25, and 30 years. The temporal order analysis suggests Rebalance increases shortfalls and, in fact, is harmful.
2. The larger the proportion of stocks to bonds in the portfolio, the longer the

Table 6: Regression Results of Percent of Initial Balance on Multiple Dummy Variables

	Bootstrap Regression	
	Coefficients	P-Value
Intercept	444.63	0.0000
γ_3	91.37	0.0000
γ_5	–89.14	0.0000
γ_6	–165.71	0.0000
γ_7	–230.56	0.0000
λ_3	–145.11	0.0000
λ_4	–74.64	0.0000
λ_6	81.72	0.0000
λ_7	161.25	0.0000
λ_8	232.83	0.0000
R	–168.75	0.0000
BF	71.60	0.0000
SF	–158.79	0.0000
HF	–63.47	0.0000

Number of Observations = 150
R^2 = .97 SEE = 37.87

portfolio tends to last. This conclusion is in agreement with previous research on the topic.

3. The probability of shortfall with either Rebalance or Bonds First is relatively low (10 percent or less) as long as withdrawals do not exceed 4 percent of the starting portfolio value and stock exposure is less than 70 percent ($\lambda < 0.7$.) Shortfall risk increases as the withdrawal rate, γ, increases. This is in line with findings of Cooley et al. (1999, 2003), Bengen (2004), and others.
4. The shortfall regression for the bootstrap in Table 2 as well as the shortfall regression for the temporal order analysis in Table 4 suggest that Bonds First is the preferred harvesting strategy to minimize shortfalls. Not only did Bonds First provide significantly fewer shortfalls for the 30-year period under study, but also for periods of 15, 20, and 25 years.

In the percent-of-initial-balance-remaining (PIBR) analysis,

1. Rebalance is shown to be the least effective harvesting method and Bonds First the best for maximizing PIBR.
2. When the withdrawal strategy does *not* run out of money before the 30-year period has elapsed, harvesting Bonds First with no rebalancing provides the greatest PIBR. This outcome is consistent with the conclusions reached in Spitzer and Singh (2006.)

The conclusions on the efficacy of depleting bonds first (and hence not rebalancing) may seem risky. Suppose that an investor has a 50/50 stock/bond portfolio at the onset of retirement. If the bonds are withdrawn first, over a period of, say, 12 years, the stock component of the portfolio has had 12 years to grow undisturbed by withdrawals. Even though stocks are more volatile, this 12-year hiatus could have allowed a significant increase in the stock part of the portfolio. *Both of the shortfall analyses demonstrate that one is no more likely to run out of money using this strategy than if one rebalances.* While one can argue that the retiree's portfolio will get more and more volatile over time as fewer and fewer bonds remain, the evidence does not suggest that shortfall is less likely with rebalancing. It then becomes a matter of how risk is viewed—as portfolio volatility (return variance) or as shortfall risk.

The PIBR analysis suggests that the Bonds First withdrawal strategy is likely to leave a larger portfolio at the end of 30 years than rebalancing would. A reasonable recommendation to a retiree would be to first harvest bonds and then harvest stocks. This strategy provides outcomes at least equivalent to rebalancing with respect to shortfalls and has the added benefit of a potentially larger estate. An advisor, however, still needs to be cognizant of the behavioral aspects associated with portfolio volatility.

The current trend in retirement planning uses life-cycle funds, which change portfolio asset allocation as a function of the age of the retiree—the older the retiree, the smaller the proportion of the portfolio in stocks. If minimizing shortfall risk is the retiree's ultimate goal, these results suggest that the life-cycle strategy—at least during the withdrawal phase—needs additional empirical justification. Bonds First (the withdrawal strategy that seems to result in the fewest shortfalls) will undoubtedly result in a portfolio with a *rising* stock proportion over time—contrary to the practice of the life-cycle funds.

JFP

Endnotes

1. Bootstrap simulations take random samples (with replacement) from available real-world data. This method contrasts with Monte Carlo simulations, which generate data from statistical distributions with known parameters.
2. We would like to thank Dale Domian for pointing out a small methodological problem in an earlier version of this paper.
3. The bootstrap and the historical sequencing provide two distinct facets to the investigation: the bootstrap allows repeated sampling from a relatively small population from which statistically valid conclusions may be drawn, while the temporal sequence can (limitedly) account for serial correlations within the data. In the bootstrap simulation, one of the 78 years is randomly selected each time; rates of return that actually occurred are selected and the difference between those rates is historically correct. But the number of possible *sequences* (orderings) of the rates is extremely large and the sequence will influence the success or failure of the withdrawal process. Since sampling is with replacement, there are 78 ways to select the first year, 78 ways to select the second year, and so on. All told there are $78^{30} \approx 5.8 * 10^{56}$ different 30-year sequences. The bootstrap portion of the study looks at 10,000 such sequences for each of the 150 possible conditions, while the temporal portion concentrates on the 49 sequences that actually occurred.
4. The average is not a very reliable measure for these data and certainly not something that represents an expected outcome. Each distribution of ending portfolio balances is replete with a large number of zero values and is noticeably right-skewed (a long tail to the right.) A few very large values for the ending portfolio balance can inflate the average balance to unrealistic levels. Retirees may expect to attain the median amount half the time, but few individuals will attain the percent of initial balance remaining (PIBR) shown in Table
5. As an example, in Table 5 for $\lambda = 0.5$, $\gamma = 7\%$, with Low First harvesting, the PIBR is 218 percent—the ending portfolio is 2.18 times larger than the starting portfolio. However, in Table 1 for the same λ, γ, and harvest conditions, 55 percent of the portfolios run out of money before 30 years have elapsed. It is easy to see that the median portfolio balance in this example (at the 50th percentile) must be zero! More than half the time, there will be *nothing* left

in the portfolio under these circumstances. The PIBR is suggestive, but should not be viewed as reliable.

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