The Asset Allocation Debate: A Review and Reconciliation
by Yesim Tokat, Ph.D.; Nelson Wicas, Ph.D.; and Francis M. Kinniry, CFA

Executive Summary

- This paper reviews several aspects of the asset allocation debate and offers observations to reshape or provide a fresh perspective.
- The first area of exploration is the debate over the well-known 1986 study by Brinson, Hood, and Beebower, in which they contend that the changes in portfolio return variations over time can be explained by static index implementation of asset allocation versus active management. This is measured by the time-series R-squared.
- Critics have focused on the degree to which actual returns can be explained by asset allocation versus active management. This is measured by the cross-sectional R-squared.
- The paper contends that actual and policy returns may have a very high time-series R-squared and, at the same time, a very low cross-sectional R-squared, resulting in very different overall returns.
- The paper also confirms that the nature of the samples has influenced past results. The magnitudes of time-series and cross-sectional R-squared is lower for portfolios that engage in a greater degree of active management and which are less diversified.
- The debate should be refocused from R-squared to what really matters to investors: whether active management can increase risk-adjusted return. The paper finds that, on average, active management reduces return and increases volatility.
- Several proponents have suggested replacing static asset allocations with dynamic allocations, which change with expected returns and capital market opportunities. Although this premise is sound, dynamic asset allocation can enhance portfolio performance only if investors have the ability to consistently predict expected returns in financial markets.

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This paper reviews several aspects of the asset allocation debate and offers observations to reshape or provide a fresh perspective on the debate. We start with the most widely discussed debate: the determinants of return variation (the focus of Brinson, Hood, and Beebower's well-known 1986 study) versus the determinants of return (the heart of Jahnke's 1997 critique of Brinson). We explore the impact of the sample used in the Brinson study on the results of the study and the implications for an investor with a broader set of investment options. We then suggest a refocusing of the debate to those matters critical to investors, namely whether active management increases return or decreases risk. Finally, we review the current debate over dynamic versus static asset allocation policies and conclude that the market-timing component...
of dynamic allocation makes it problematic.

**Literature Review**

The asset allocation debate emerged in response to Brinson, Hood, and Beebower's 1986 paper, "Determinants of Portfolio Performance," in which the authors concluded that a portfolio's asset allocation, or policy return, is the primary determinant of total return variation, with security selection and market timing playing minor roles. In their study of 91 large U.S. pension plans, the authors found that policy return explained 93.6 percent of the variation in actual plan returns. In a follow-up to this paper, Brinson, Singer, and Beebower (1991) generally confirmed these results with a slightly lower number: 91.5 percent. In the past decade, some research has confirmed the original study's conclusions (Ibbotson and Kaplan 2000; Vanguard 2003).

Other authors, most notably William Jahnke (1997), have criticized the study, or more accurately its interpretation by the investment industry, and raised doubts about its applicability to general investors. Jahnke argues that the volatility of portfolio returns over time is unimportant to investors. Investors care about actual returns and the range of possible investment outcomes at the end of their time horizons. We compare and reconcile the two views based on our analysis of a sample of balanced mutual fund returns.

Another important influence on the asset allocation debate is the nature of the sample. As Ankrim and Hensel (2000) point out, the importance of asset allocation depends upon on the "base case" asset allocation and the policy allocation studied. For example, these authors found that policy portfolios that are less "index-like" or more variable will yield different results from those that are more "index-like" and less variable. It follows that Brinson's (1986) results are a function of the broadly diversified nature and limited active management of the pension fund portfolios studied. Brinson (1986) found that pension funds were exposed to a high level of systematic risk, resulting in a strong relationship between the funds' actual and asset allocation (policy) return. Ibbotson and Kaplan (2000) found similar results. Our results support these findings.

We suggest a refocusing of the debate away from R-squared measures to the critical issue for investors: the value (or nonvalue) of active management in improving returns and reducing volatility (see Statman, 2000, for a similar view). The most important contribution of Brinson et al. (1986) has been the attribution of a portfolio's total return to indexed static asset allocation policy, security selection, and market-timing components. Their study showed that, on average, pension funds have not been able to add significant value above their indexed static policy returns through market timing or security selection. This result is consistent with the outperformance of indexing in equity and bond markets (Carhart 1997, and others). Despite the large potential influence of security selection and market-timing strategies on portfolio returns, the amount of skill required to justify active management is very high (Kritzman and Page 2003). Active returns tend to be unstable and unpredictable over time (Carhart). We review these results and present our findings on the value of active management.

In a related debate, several authors have recently questioned whether investors should implement dynamic rather than static asset allocation policies (Jahnke and Bernstein 2003, Foley 2004). Jahnke was the first to point out that the investment industry's interpretation of the 1986 Brinson studyÑnamely its conclusion that an indexed static asset allocation policy is the optimal approach for investorsÑis misinterpreted. In the industry, Brinson's conclusions were typically used to support focusing on getting the asset allocation right without much regard to funds' performances or costs. In addition, Jahnke noted that static allocations should be related directly to a client's specific circumstances or long-term financial goals. This conclusion is rooted in financial theory and not controversial.
More recently, however, others have suggested that dynamic asset allocation policies reflect changing expected returns and capital market opportunities. (Bernstein, Foley, and Jahnke 2004). Dynamic policy asset allocation strategies require asset return predictability. When asset returns are predictable, optimal asset allocation policy involves market timing and inter-temporal hedging (Campbell and Viceira 1999). Yet asset-return predictability studies (for example, Goyal and Welch 2004, and Campbell and Thompson 2004) show that in-sample predictive ability of financial and economic variables strongly deteriorates in out-of-sample forecasts. Goyal and Welch, for example, found that the equity premium was not predictable for practical purposes, and that any belief about whether the stock market is now too high or too low was to be based on theoretical prior beliefs, not on the variables they explored. In other words, the variables they explored would not have helped a real-world investor predict returns consistently because they worked only when applied to certain historical periods. The Brinson study raises additional doubts about the wisdom of dynamic asset allocation. We review their results and present our findings.

Data

We analyzed balanced, asset allocation, and total return open-end funds from the University of Chicago CRSP Survivor-Bias Free U.S. Mutual Fund Database. The data include monthly net returns, annual allocations to asset classes, and some fund characteristics such as expense ratios and turnover.

We selected funds using several criteria. First, we required each fund to hold over its lifetime more than 20 percent of both average long-run equity and bond allocations. Second, we excluded funds with more than 5 percent of their assets devoted over their lifetime to an asset class other than stocks, bonds, and cash. Among the remaining funds, we selected total return, income, asset allocation, and traditional balanced funds based on CRSP fund categorizations. If a fund return for a single month was missing, we excluded that month from the analysis.

To ensure statistical reliability of style analysis, we required funds to have at least 36 months of returns. While this introduced omission bias from excluding funds that ceased reporting, it diminished the incubation bias from the private histories of new funds. These opposing factors produced a net effect close to zero.

Empirical Methodology

To determine the relative performance of asset allocation policy and active management, we distinguished between a portfolio's policy return—what it would have earned if it had simply re-created its policy allocation with unmanaged index funds—and its actual return—the real-world return that reflects a fund's execution of active strategies. We calculated a fund's policy return through indirect empirical methods because, in a universe of actively managed funds, the policy return is, by definition, not observed in the actual returns. Our empirical and quantitative analysis included six primary steps:

1. Style analysis, which allowed us to infer the funds' policy allocations
2. Simple calculation of policy returns using asset-class benchmarks and policy weights inferred from style analysis
3. Time-series analysis of the regression of the funds' actual returns against their policy returns over time which gave us the time-series R-squared
4. Calculation of the ratio of a fund's actual average return to the average return of its policy allocation
5. Calculation of the ratio of a fund's actual volatility to the volatility of its policy allocation
6. Cross-sectional analysis—a regression of the funds’ actual returns against their policy returns in a given period—which gave us the cross-sectional R-squared. (For details of these calculations, see the appendix.)

For stock market returns, we used the Wilshire 5000 Total Market Index from 1971 to 2003 and the Standard & Poor's 500 Index from 1962 to 1970. For bond market returns, we used the Lehman Brothers U.S. Aggregate Index from 1976 to 2003, the Citigroup High Grade Corporate Index from 1969 to 1975, and the S&P High Grade Corporate Index from 1962 to 1968. For the returns on cash investments, we used the Citigroup Three-Month U.S. Treasury Bill Index from 1978 to 2003, and the Three-Month Treasury Bill rate from 1962 to 1977.¹

**Time-Series or Cross-Sectional R-Squareds: What Do They Mean to Investors?**

The Brinson study represents a time-series analysis of the effect of asset allocation on performance. The methodology compares the performance of a policy, or long-term, asset allocation represented by appropriate market indexes with the actual performance of a portfolio over time. This approach finds that most of a portfolio’s return variability—the change in returns over time or return patterns—can be attributed to its policy asset allocation return variability. Active investment decisions—market timing and security selection—have relatively little impact on return patterns.

This statement is not controversial, at least not in a universe of broadly diversified pension funds with limited market timing. But return patterns are not the same thing as actual returns. A portfolio may end up with very different wealth amounts at the end of the investment horizon, depending on which fund or funds were selected. For example, Brinson's approach might show that the return patterns over time of two funds, each with 60 percent stock/40 percent bonds, is explained primarily by their asset allocation. What the Brinson methodology does not reveal is that these two funds can have very different total returns, reflecting the results of the active decisions made in each portfolio.

As illustrated in Figure 1, idiosyncratic risks and differential exposure to systematic risk factors (factor or tactical bets) can create significant performance differences, resulting in a low cross-sectional R-squared when actual returns are regressed on policy returns in a given period. In other words, policy returns may not explain a large portion of actual returns. At the same time, the time-series R-squared of those same funds may be quite high. In other words, policy return variation over time may explain a large portion of actual return variation over time. In the figure’s hypothetical example, return patterns are very similar, yet actual returns and policy returns are not the same.
Our study of asset allocation, total return, and traditional balanced mutual funds supports this intuition. Table 1 displays the results of the study. The first column represents the relationship between the actual and policy return patterns, the time-series R-squared. It shows that, on average, returns of these balanced funds tend to move in tandem with the markets over time. The second column displays a much lower R-squared, the cross-sectional R-squared. These figures are at the heart of the “cross-sectional” critique of the Brinson study. As Jahnke found, the percentage of actual returns explained by policy allocations can be much lower than the percentage of actual return variation explained by policy allocations. Our results show that balanced funds’ policy allocations can explain less than 20 percent of their actual monthly returns.² This suggests that although balanced fund returns moved in tandem with broad markets over time, the actual returns have been different from one another. These actual returns reflect each fund's idiosyncratic risks, risk-factor bets, costs, luck, and investment decisions.³
The Impact of Sample Population on Time-Series and Cross-Sectional R-Squareds

The magnitude of time-series and cross-sectional R-squareds depends on the portfolios analyzed. All broadly diversified, passively managed portfolios are exposed to the systematic (undiversifiable) risk factors in financial markets, such as business cycles and interest rates. An assessment of what drives the performance of a broadly diversified portfolio over time is likely to find a strong relationship between the portfolio and market returns. Likewise, if policy portfolios have asset class weightings similar to the broad market with limited active management, the policy portfolios should have returns and return patterns similar to the broad market.

Consider a balanced portfolio that holds one stock and one bond. Changes in the price of each security would be influenced by the general movements of the stock and bond markets, producing a relatively high time-series R-squared between return patterns of the portfolio and stock and bond markets. It's likely, however, that the total return produced by the broad stock and bond markets and the total return of the two-security portfolio would be very different, leading to a low cross-R-squared between the actual portfolio return and the market. On the other hand, if a group of balanced funds had a larger number of broadly diversified holdings and simply implemented their static policy allocations with index funds, both the time-series and cross-sectional R-squareds would theoretically be 100 percent (policy performance would explain performance variation across funds as well as over time).

The high time-series R-squared of the Brinson study is a result of the broadly diversified nature and limited active management of pension fund portfolios. For instance, in Brinson (1986), the lowest time-series R-squared is 75.5 percent, indicating that pension funds closely followed their indexed static asset allocation policies. Updates of the study (Ibbotson and Kaplan 2000, and Vanguard 2003) found that while balanced mutual funds are also typically broadly diversified, they tend to be more active than pension funds, leading to lower time-series and cross-sectional R-squared. For instance, fifth percentile time-series R-squared is 46.9 percent for Ibbotson and Kaplan's balanced mutual fund sample. In our sample, which

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<th>% of Actual Return Variation Explained by Policy Return Variation</th>
<th>% of Actual Return Dispersion Explained by Policy Return Dispersion</th>
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<tbody>
<tr>
<td>Average</td>
<td>81.61%</td>
<td>18.86%</td>
</tr>
<tr>
<td>Median</td>
<td>85.48%</td>
<td>14.97%</td>
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</table>

Note: Monthly return dispersion. Results are similar for three-year return dispersion.

Notes: Balanced fund data includes total return, income, asset allocation, and traditional balanced funds. Multiple share classes of the same fund are aggregated by fund asset size weighting returns. Balanced fund policy benchmarks are assigned using style analysis over five-year rolling periods (requiring minimum three years of data).

Source: University of Chicago CRSP Survivor-Bias Free U.S. Mutual Fund Database Guide; authors' calculations.
includes total return, asset allocation, and balanced funds, we find that the lowest time-series R-squared is 30.7 percent.4 These results suggest that the magnitude of time-series and cross-sectional R-squared is a factor of the degree of active management in the portfolio.

What Matters Most to Investors: Return and Risk

What has been overlooked in this debate is that the ultimate concern of an investor is not the time-series or cross-sectional R-squared, but whether active management can increase return without increasing the risk of a portfolio. Although greater degrees of active management reduce both time-series and cross-sectional R-squared, it would not necessarily increase performance. The initial Brinson study provided the framework for addressing this issue, and our analysis supports these results. Table 2 shows that actively managed balanced mutual funds, on average, have detracted from performance and increased portfolio volatility relative to their indexed static policy portfolios. The first column shows policy returns as a percentage of actual returns. Our results show that policy return contributed more than 100 percent of actual returns, and therefore, that the contribution of active management to actual returns was negative. The second column shows policy volatility as a percentage of actual return volatility. Our results show that policy volatility was smaller than actual return volatility. Overall, our results show that the policy portfolio produced higher returns with less risk, on average.


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<th>Policy Return as % of Actual Return</th>
<th>Policy Volatility as % of Actual Return Volatility</th>
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<tbody>
<tr>
<td>Average Median</td>
<td>122.13% 105.94%</td>
<td>90.23% 92.15%</td>
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Notes: 13 funds that had a negative average actual or policy return were excluded from this table because the ratio of policy return to actual return is not meaningful. Only 3 out of these 13 funds outperformed their policy benchmark. In this sense, this table understates the outperformance of the policy portfolio. Before the hypothetical cost of implementing the policy portfolio is deducted, using all 227 funds, actual portfolio underperformed the policy portfolio, on average, by five basis points in any given month, and policy portfolio outperformed actual portfolio for 64.76 percent of the funds. One fund, which had the ratio of policy return to actual return of 297 percent, was excluded from the average ratio of policy return to actual return reported in the table because this outlier skew the average higher. Policy portfolio is assumed to have a cost of 2 basis points a month (approximately 25 basis points annually).

Source: University of Chicago CRSP Survivor-Bias Free U.S. Mutual Fund Database Guide; authors' calculations.

Despite the averages, active management has created meaningful performance differences among funds. Figure 2 illustrates that from 1966 to the present, the average median net excess return of balanced funds over their indexed static policy benchmarks has been negative. The results have been similar over shorter time frames. Yet Figure 2 also shows that when funds are ranked based on their rolling five-year net excess returns, there can be large differences between the top and bottom quartiles. Confirming Jahnke's criticism, the return difference between the top and bottom 25 percentile funds
The goal of active management is to increase the risk-adjusted return of a portfolio. Yet although active management can create significant differences in performance among funds, the amount of skill required to justify active management is very high (Kritzman and Page 2003). As illustrated in Figure 3, 61 percent of balanced funds underperformed their policy portfolio on an annual basis over the past ten years. About 64 percent underperformed their policy portfolios over three and five years. Active management around the static index implementation of asset allocation policy entails greater opportunities and risks, which, on average, are not compensated.
Should Asset Allocation Policy Be Static or Dynamic?

A recent asset allocation debate has called into question the wisdom of establishing a static long-term policy allocation (Bernstein 2003, Foley 2004, Jahnke 2004). Investors determine their asset allocation policy based on their risk tolerance, financial goals, time horizon, sources of non-market wealth (such as earned income), and risk premiums for asset classes. Any one of these variables can change, potentially prompting a change in an investor's asset allocation policy. Some changes are easy to gauge—a change in time horizon or financial goals, for example—allowing for a relatively simple adjustment to the policy allocation. Other changes are harder to detect, such as variations in the expected returns and risk premiums.

Initially, criticism of static-allocation approaches centered on the fact that static allocations rarely related directly to a client's specific circumstances or long-term financial goals (Jahnke 1997). It is clear from financial theory and practical experience that investors' asset allocation choices should be linked with their specific circumstances or long-term financial goals. More recently, several authors have issued a more profound challenge to the concept of a static policy asset allocation. These researchers are asking whether investors should change their asset allocation policies dynamically in response to changing expected returns and capital market opportunities—that is, time the market (Bernstein, Foley, and Jahnke 2004). The logic is that expected returns are not static, so asset allocation should not be static. This view assumes
some predictable variation in asset class returns. The view is supported with evidence of mean reversion within asset classes, as opposed to the random walk theory of asset returns. If mean reversion exists, after a period of strong or weak performance, at some point, it would be rational for an investor to change expectations. At that point, asset allocation policies should change accordingly.

Although the critics' premise is sound, the implementation of dynamic asset allocation is challenging. Only if investors have the ability to consistently predict expected returns in financial markets can dynamic or tactical asset allocation enhance portfolio performance. Asset-return predictability studies show that in-sample predictive ability of financial and economic variables strongly deteriorates in out-of-sample forecasts (for example, Goyal and Welch 2004, and Campbell and Thompson 2004). These studies considered a myriad of signals, the economic and financial variables traditionally used to predict returns, and found most signals that produced excess returns in the past did not do so in the future. As these studies found, signals can produce significant excess returns for one period, but not in the next, which means they cannot consistently predict future return. In other words, it is difficult to consistently benefit from them.

The Brinson study raises additional doubts about the wisdom of dynamic asset allocation. If we assume that pension funds in Brinson's study changed their asset allocation policies in response to changing market conditions (rather than in response to funding concerns), Table 3 indicates that even before management costs, active asset allocation, on average, has detracted from the performance of pension funds from 1974 to 1987. This finding underscores the difficulty of timing markets. Yet it is important to recognize that some pension funds have done better and others have done worse than their policy performance.

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<tr>
<th>Table 3: Historical Returns from Market Timing and Security Selection</th>
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<tr>
<td>Market Timing</td>
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<td>Security Selection</td>
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<tr>
<td>Other</td>
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<td>Total Active Return</td>
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Because the impact of active management tends to be less stable and less predictable than the impact of asset allocation choice, our recommendation is to select asset allocations appropriate to investors' unique circumstances, and construct a broadly diversified portfolio with limited market timing. To the extent that active management plays a role in a portfolio, investors should select active funds where impediments to skill, such as costs, are lower. Asset allocation remains the primary determinant of return in portfolios made up of index or broadly diversified funds with limited market timing.

**Conclusion**
Based on our review of the asset allocation debate, we have several findings. First, the source of the largest debate is simply a different focus: return variation over time versus return differences among funds.

Second, past study results are strongly dependent on the study sample. Samples comprising diversified portfolios with limited active management produce higher time-series and cross-sectional R-squared than those with less broadly diversified, actively managed portfolios.

Third, we refocus the debate away from the R-squared to what is really the ultimate concern in the active/passive decision: whether active management can increase the returns or decrease the risks of a portfolio. We find that, on average, active management reduces a portfolio's returns and increases its volatility compared with a static index implementation of the portfolio's asset allocation policy. Nonetheless, active management creates an opportunity for a portfolio to outperform appropriate market benchmarks.

Fourth, dynamic asset allocation strategies that attempt to change asset allocations with changing expected returns for the market—that is, time the market—can be problematic. Dynamic or tactical strategies of this nature, of course, will only enhance returns if investors have the ability to consistently predict expected returns in financial markets. Studies of predictability have shown that many of the traditionally used signals of market performance are, in fact, not consistently predictive when used in real time.

In summary, unless there is a strong belief in the ability to select active managers who will deliver higher risk-adjusted net returns, investors' focus should be on the asset allocation choice and its implementation using broadly diversified, low-cost portfolios with limited market timing.

Endnotes

1. When a series was not available as far back as we wanted, we backfilled it with a close proxy.
2. The cross-sectional R-squared for rolling five-year returns, which is not reported here, is also less than 20 percent.
3. Since misinterpretation of the R-squared is at the heart of the asset allocation debate, it was a focus of our study. We did not explore beta because it was not pertinent to the debate.
4. We find that the fifth percentile time-series R-squared is 52.8 percent in our mutual fund sample.
5. Since the main point of Figure 2 is simply to illustrate the return differences, we have not tested the statistical significance of the differences.
6. Note that active managers may have somewhat different styles, market caps, or credit exposures than the broad benchmarks used in this study. These differential factor bets may influence the percentage of balanced funds underperforming their benchmarks figures reported in Figure 3.

References


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**Appendix**

1. Estimation of Policy Allocation Using Style Analysis

\[ \pi_{\text{policy}} = W_{\text{Stock}} \pi^S + W_{\text{Bond}} \pi^B + W_{\text{Cash}} \pi^C - \text{cost}. \]
The policy weightings, or asset allocation, for each fund were estimated by performing returns-based style analysis over the rolling five-year history of the fund. Style analysis (Sharpe 1988) is a statistical method for inferring a fund's effective asset mix by comparing the fund's returns with returns of asset-class benchmarks. Style analysis is a popular attribution technique because it does not require tabulating the actual asset allocation of each fund for each month over time. Rather, style analysis facilitates return attribution by regressing the return of the fund against the returns of asset-class benchmarks. The following regression is estimated:

\[ r_{t}^{\text{fund}} = \alpha + w_{\text{Stock}} r_{t}^{S} + w_{\text{Bond}} r_{t}^{B} + w_{\text{Cash}} r_{t}^{C} + \varepsilon_{t}, \]

where

- \( w_{\text{Stock}} \) is the policy allocation to stocks
- \( w_{\text{Bond}} \) is the policy allocation to bonds
- \( w_{\text{Cash}} \) is the policy allocation to cash
- \( r_{t}^{S} \) is the return on equity benchmark in period \( t \)
- \( r_{t}^{B} \) is the return on bond benchmark in period \( t \)
- \( r_{t}^{C} \) is the return on cash benchmark in period \( t \)
- \( \alpha \) is the excess return on the fund that cannot be attributed to the returns on benchmarks
- \( \varepsilon_{t} \) is the residual that cannot be explained by the asset class returns.

For our purposes, style analysis requires not only that asset-class weight parameters sum to 1, but also that each asset-class weight is positive (no short sales).

3. Time-Series Regression of Actual Returns Against Policy Returns

To compare variation in the policy and actual returns, we calculated an R-squared for each fund by regressing its actual return against its policy return over time:

\[ r_{t}^{\text{fund}} = \alpha + \beta r_{t}^{\text{policy}} + \varepsilon_{t}. \]

where

- \( \alpha \) is the excess return on the fund that cannot be attributed to policy return
- \( \beta \) is the sensitivity of changes in fund return to changes in policy return
- \( \varepsilon_{t} \) is the residual, or error term, that cannot be explained by policy return.

This time-series regression reveals what portion of the pattern of returns over time is explained by policy returns. We ran the time-series regression for each fund over the period stated.
returns of asset class benchmarks in the following way:

Appendix (continued)

R-squared presented in our results is based on the average R-squared of each fund.

4. The Ratio of the Average Policy Return to the Average Actual Return

The fund policy return as a percentage of the actual return is

\[ \frac{\text{average policy return}}{\text{average actual return}} \]

Using the standard way to calculate sample mean, averages are calculated as

\[ \left( \frac{1}{T} \sum_{t=1}^{T} r_{t}^{\text{policy}} \right) \left( \frac{1}{T} \sum_{t=1}^{T} r_{t}^{\text{fund}} \right) \]

When the average policy return is greater than the average actual return, this ratio is greater than 100 percent.

5. The Ratio of Policy Volatility to Actual Volatility

The fund policy volatility as a percentage of the actual return volatility is

\[ \text{standard deviation of the policy return} / \text{standard deviation of the actual return} \]

The standard way to calculate sample standard deviation is

6. Cross-Sectional Regression of Actual Returns Against Policy Returns

To compare what portion of actual returns can be explained by policy returns, we calculated an R-squared by regressing the actual returns against the policy returns of all funds at a given point in time (Ibbotson and Kaplan 2000):

\[ r_{t}^{\text{fund}} = \alpha + \beta r_{t}^{\text{policy}} + \epsilon_{t} \]

where

- \( \alpha \) is the excess return on the fund that cannot be attributed to policy return
- \( \beta \) is the sensitivity of changes in fund return to changes in policy return
- \( \epsilon_{t} \) is the residual, or error term, that cannot be explained by policy return

This cross-sectional regression reveals what portion of actual returns in one period is explained by policy returns. We ran the cross-sectional regression for each month over the period stated. The aggregate R-squared presented in our results is based on the average of the monthly regression calculations. See Ibbotson and Kaplan (2000) for a similar approach.

Endnote to Appendix
\[
\left( \frac{1}{T-1} \sum_{t=1}^{T} \left[ r_{t,\text{policy}} - \text{avg}(r_{t,\text{policy}}) \right]^2 \right)^{\frac{3}{2}} \left( \frac{1}{T-1} \sum_{t=1}^{T} \left[ r_{t,\text{fund}} - \text{avg}(r_{t,\text{fund}}) \right]^2 \right)^{\frac{3}{2}}
\]

When policy volatility is smaller than actual return volatility, this ratio is less than 100 percent.

1. This approach allows us to account for long-term policy shifts that reflect changes in a fund's risk tolerance or assessment of long-term changes in risk premiums. Any short-term deviation from the five-year policy is considered active management.